MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2015/2016

BST3254 - MONTE CARLO SIMULATION TECHNIQUES

(All sections / Groups)

5 MARCH 2016 9.00 a.m – 11.00 a.m (2 Hours)

INSTRUCTIONS TO STUDENTS

- 1. This question paper consists of FOUR (4) printed pages excluding the cover page, statistical formulae and tables.
- 2. Answer ALL questions.
- 3. Only non-programmable calculator is allowed to be used in this examination.
- 4. Statistical tables are attached at the end of the question paper.

Question 1 (25 Marks)

The time between arrivals (in minutes) at a counter of a cafeteria in Putrajaya and the service time (in minutes) follow the distribution given in the tables below.

| Time between arrivals | Probability |
|-----------------------|-------------|
| 1 | 0.15 |
| 2 | 0.25 |
| 3 | 0.35 |
| 4 | 0.25 |

| Service time | Probability |
|--------------|-------------|
| 2 | 0.20 |
| 3 | 0.50 |
| 4 | 0.30 |

(a) Use the random numbers provided to simulate the activity of the first five arrivals. Your simulation table should include time of arrival, service start and end times, time spent waiting in line and idle times. Assume that the counter opens at 8.15 a.m. and the first arrival after this based on the first interarrival time generated.

Random numbers for arrivals: 07, 57, 84, 00, 32 Random numbers for service time: 60, 17, 55, 59, 73

(20 marks)

(b) At what time did the third customer leave the system?

(2 marks)

(c) Compute the average waiting time the customer spends in a queue.

(3 marks)

Continued...

Question 2 (25 Marks)

A marketing manager of a multinational company is planning a questionnaire survey to assess the customer satisfaction level towards its product. He has identified the following tasks to be carried out in order to conduct the survey successfully:

| | Immediate | | Duration (Days) | |
|----------|--------------|------------|-----------------|-------------|
| Activity | Predecessors | Optimistic | Most Likely | Pessimistic |
| A | ~ | 4 | 5 | 6 |
| В | - | 8 | 12 | 16 |
| C | A | 4 | 5 | 12 |
| D | В | 1 | 3 | 5 |
| Е | D,A | 2 | 2 | 2 |
| F | В | 3 | 4 | 5 |
| G | C,E,F | 10 | 14 | 18 |
| Н | G | 18 | 20 | 34 |

(a) Determine the activity time (t) for each task.

(5 marks)

(b) Represent the design involved in the form of an appropriate network of activities.

(3 marks)

(c) Identify the critical path and the expected completion time of the project.

Determine the earliest start time, latest start time, earliest finish time, latest finish time and slack time.

(17 marks)

Continued...

Question 3 (25 Marks)

(a) Generate exponential variates X_i with mean 5 for the following random numbers:

0.65

0.73

0.37

0.44

0.59

0.29

(7 marks)

(b) The activity times (in seconds) for a bagging operation were recorded as follows:

| 11.3 | 8.2 | 16.8 | 10.3 |
|------|------|------|------|
| 7.2 | 8.6 | 15.2 | 9.6 |
| 12.5 | 7.4 | 8.3 | 11.1 |
| 14.3 | 11.1 | 14.5 | 11.8 |
| 12.8 | 12.3 | 10.7 | 9.5 |
| 13.8 | 10.2 | 14.9 | 16.3 |
| 15.2 | 7.7 | 12.9 | 12.4 |
| 13.5 | 11.0 | 14.3 | 16.9 |
| 9.2 | 13.2 | 7.5 | 13.2 |
| 16.3 | 14.4 | 15.1 | 10.7 |
| | | | |

Use the chi-square test to test the hypothesis that the activity times are uniformly distributed. Let the number of intervals be k = 5. Use the level of significance $\alpha = 0.10$.

(18 marks)

Continued...

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Question 4 (25 Marks)

(a) Consider the following pdf for a random variable X,

$$f(x) = \begin{cases} (x+2)/12, & 1 \le x \le 3\\ \sqrt{x-2}/14, & 3 < x \le 6\\ 0, & \text{elsewhere} \end{cases}$$

Use the inverse-transformation technique to show that the random variate generator of X is

$$X = \begin{cases} -2 + \sqrt{24R + 9}, & 0 \le R \le 2/3 \\ 2 + (21R - 13)^{2/3}, & 2/3 < R \le 1 \end{cases}$$

(10 marks)

(b) Consider the interarrival and service times (in minutes) provided as below:

| Interarrival times | 2 . | 4 | 6 | 8 | 10 |
|--------------------|-----|---|---|---|----|
| Service times | 1 | 3 | 6 | 5 | 4 |

(i) Assuming that the starting clock is 0, compute the arrival and departure times for 5 customers.

(5 marks)

(ii) Prepare Discrete Event Simulation (DES) table for this system until the clock reaches times 18. The stopping event will be at time 45.

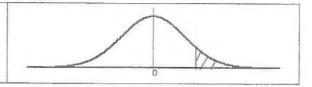
(10 marks)

End of Page

STATISTICAL TABLES

| Freedom | | | |
|------------------|---------------------|---------------------|---------------------|
| (N) | $D_{0.10}$ | D _{0:05} | $D_{0.01}$ |
| 1 | 0.950 | 0.975 | 0.995 |
| 2 | 0.776 | 0.842 | 0.929 |
| 3 | 0.642 | 0.708 | 0.828 |
| 4 | 0.564 | 0.624 | 0.733 |
| 5 | 0.510 | 0.565 | 0.669 |
| 6 | 0.470 | 0.521 | 0.618 |
| 7 | 0.438 | 0.486 | 0.577 |
| 8 | 0.411 | 0.457 | 0.543 |
| 9 | 0.388 | 0.432 | 0.514 |
| 10 | 0.368 | 0.410 | 0.490 |
| 11 | 0.352 | 0.391 | 0.468 |
| 12 | 0.338 | 0.375 | 0.450 |
| 13 | 0.325 | 0.361 | 0.433 |
| 14 | 0.314 | 0.349 | 0.418 |
| 15 | 0.304 | 0.338 | 0.404 |
| 16 | 0.295 | 0.328 | 0.392 |
| 17 | 0.286 | 0.318 | 0.381 |
| 18 | 0.278 | 0.309 | 0.371 |
| 19 | 0.272 | 0.301 | 0.363 |
| 20 | 0.264 | 0.294 | 0.356 |
| 25 | 0.240 | 0.270 | 0.320 |
| 30 | 0.220 | 0.240 | 0.290 |
| 35 | 0.210 | 0.230 | 0.270 |
| 35 Over 35 | 0.210 1.22 √N | 0.230 1.36 √N | 0.270 1.63 √N |

Table 1
The Upper Tail Area Under the
Standard Normal Curve



| | 1 | 1 | | | | | | | | |
|-----|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| Z | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
| 0.0 | 0.5000 | 0.4960 | 0.4920 | 0.4880 | 0.4840 | 0.4801 | 0.4761 | 0.4721 | 0.4681 | 0.4641 |
| 0.1 | 0.4602 | 0.4562 | 0.4522 | 0.4483 | 0.4443 | 0.4404 | 0.4364 | 0.4325 | 0.4286 | 0.4247 |
| 0.2 | 0.4207 | 0.4168 | 0.4129 | 0.4090 | 0.4052 | 0.4013 | 0.3974 | 0.3936 | 0.3897 | 0.3859 |
| 0.3 | 0.3821 | 0.3783 | 0.3745 | 0.3707 | 0.3669 | 0.3632 | 0.3594 | 0.3557 | 0.3520 | 0.3483 |
| 0.4 | 0.3446 | 0.3409 | 0.3372 | 0.3336 | 0.3300 | 0.3264 | 0.3228 | 0.3192 | 0.3156 | 0.3121 |
| 0.5 | 0.3085 | 0.3050 | 0.3015 | 0.2981 | 0.2946 | 0.2912 | 0.2877 | 0.2843 | 0.2810 | 0.2776 |
| 0.6 | 0.2743 | 0.2709 | 0.2676 | 0.2643 | 0.2611 | 0.2578 | 0.2546 | 0.2514 | 0.2483 | 0.2451 |
| 0.7 | 0.2420 | 0.2389 | 0.2358 | 0.2327 | 0.2296 | 0.2266 | 0.2236 | 0.2206 | 0.2177 | 0.2148 |
| 0.8 | 0.2119 | 0.2090 | 0.2061 | 0.2033 | 0.2005 | 0.1977 | 0.1949 | 0.1922 | 0.1894 | 0.1867 |
| 0.9 | 0.1841 | 0.1814 | 0.1788 | 0.1762 | 0.1736 | 0.1711 | 0.1685 | 0.1660 | 0.1635 | 0.1611 |
| 1.0 | 0.1587 | 0.1562 | 0.1539 | 0.1515 | 0.1492 | 0.1469 | 0.1446 | 0.1423 | 0.1401 | 0.1379 |
| 1.1 | 0.1357 | 0.1335 | 0.1314 | 0.1292 | 0.1271 | 0.1251 | 0.1230 | 0.1210 | 0.1190 | 0.1170 |
| 1.2 | 0.1151 | 0.1131 | 0.1112 | 0.1093 | 0.1075 | 0.1056 | 0.1038 | 0.1020 | 0.1003 | 0.0985 |
| 1.3 | 0.0968 | 0.0951 | 0.0934 | 0.0918 | 0.0901 | 0.0885 | 0.0869 | 0.0853 | 0.0838 | 0.0823 |
| 1.4 | 0.0808 | 0.0793 | 0.0778 | 0.0764 | 0.0749 | 0.0735 | 0.0721 | 0.0708 | 0.0694 | 0.0681 |
| 1.5 | 0.0668 | 0.0655 | 0.0643 | 0.0630 | 0.0618 | 0.0606 | 0.0594 | 0.0582 | 0.0571 | 0.0559 |
| 1.6 | 0.0548 | 0.0537 | 0.0526 | 0.0516 | 0.0505 | 0.0495 | 0.0485 | 0.0475 | 0.0465 | 0.0455 |
| 1.7 | 0.0446 | 0.0436 | 0.0427 | 0.0418 | 0.0409 | 0.0401 | 0.0392 | 0.0384 | 0.0375 | 0.0367 |
| 1.8 | 0.0359 | 0.0351 | 0.0344 | 0.0336 | 0.0329 | 0.0322 | 0.0314 | 0.0307 | 0.0301 | 0.0294 |
| 1.9 | 0.0287 | 0.0281 | 0.0274 | 0.0268 | 0.0262 | 0.0256 | 0.0250 | 0.0244 | 0.0239 | 0.0233 |
| 2.0 | 0.0228 | 0.0222 | 0.0217 | 0.0212 | 0.0207 | 0.0202 | 0.0197 | 0.0192 | 0.0188 | 0.0183 |
| 2.1 | 0.0179 | 0.0174 | 0.0170 | 0.0166 | 0.0162 | 0.0158 | 0.0154 | 0.0150 | 0.0146 | 0.0143 |
| 2.2 | 0.0139 | 0.0136 | 0.0132 | 0.0129 | 0.0125 | 0.0122 | 0.0119 | 0.0116 | 0.0113 | 0.0110 |
| 2.3 | 0.0107 | 0.0104 | 0.0102 | 0.0099 | 0.0096 | 0.0094 | 0.0091 | 0.0089 | 0.0087 | 0.0084 |
| 2.4 | 0.0082 | 0.0080 | 0.0078 | 0.0075 | 0.0073 | 0.0071 | 0.0069 | 0.0068 | 0.0066 | 0.0064 |
| 2.5 | 0.0062 | 0.0060 | 0.0059 | 0.0057 | 0.0055 | 0.0054 | 0.0052 | 0.0051 | 0.0049 | 0.0048 |
| 2.6 | 0.0047 | 0.0045 | 0.0044 | 0.0043 | 0.0041 | 0.0040 | 0.0039 | 0.0038 | 0.0037 | 0.0036 |
| 2.7 | 0.0035 | 0.0034 | 0.0033 | 0.0032 | 0.0031 | 0.0030 | 0.0029 | 0.0028 | 0.0027 | 0.0026 |
| 2.8 | 0.0026 | 0.0025 | 0.0024 | 0.0023 | 0.0023 | 0.0022 | 0.0021 | 0.0021 | 0.0020 | 0.0019 |
| 2.9 | 0.0019 | 0.0018 | 0.0018 | 0.0017 | 0.0016 | 0.0016 | 0.0015 | 0.0015 | 0.0014 | 0.0014 |
| 3.0 | 0.0013 | 0.0013 | 0.0013 | 0.0012 | 0.0012 | 0.0011 | 0.0011 | 0.0011 | 0.0010 | 0.0010 |
| 3.1 | 0.0010 | 0.0009 | 0.0009 | 0.0009 | 0.0008 | 0.0008 | 0.0008 | 0.0008 | 0.0007 | 0.0007 |
| 3.2 | 0.0007 | 0.0007 | 0.0006 | 0.0006 | 0.0006 | 0.0006 | 0.0006 | 0.0005 | 0.0005 | 0.0005 |
| 3.3 | 0.0005 | 0.0005 | 0.0005 | 0.0004 | 0.0004 | 0.0004 | 0.0004 | 0.0004 | 0.0004 | 0.0003 |
| 3.4 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0002 |
| 3.5 | 0.0002 | 0.0002 | 0.0002 | 0.0002 | 0.0002 | 0.0002 | 0.0002 | 0.0002 | 0.0002 | 0.0002 |
| 3.6 | 0.0002 | 0.0002 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 |
| 3.7 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 |
| 3.8 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 |
| 3.9 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 4.0 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |

Table A.7 Critical Values for Chi-Squared Distributions

| | χ_{ν}^2 density curve |
|---|------------------------------|
| | Shaded area = a |
| | |
| 0 | |
| V | _ Xa,v |

| | | | | | α | | $-\chi_{\dot{\alpha},\nu}$ | | | |
|----------|-----------------|--------|--------|--------|----------|--------|----------------------------|------------------|--------|--------|
| W | .995 | ,99 | .975 | .95 | .90 | .10 | .05 | .025 | .01 | .005 |
| 1 | 0.000 | 0.000 | 0.001 | 0.004 | 0.016 | 2.706 | 3.843 | 5.025 | 6.637 | 7.882 |
| 2 | 0.010 | 0.020 | 0.051 | 0.103 | 0.211 | 4.605 | 5.992 | 7.378 | 9.210 | 10.597 |
| 3 | 0.072 | 0.115 | 0.216 | 0.352 | 0.584 | 6.251 | 7.815 | 9.348 | 11,344 | 12.837 |
| 4 | 0.207 | 0.297 | 0.484 | 0.711 | 1.064 | 7.779 | 9.488 | 11.143 | 13.277 | 14.860 |
| 5 | 0.412 | 0.554 | 0.831 | 1.145 | 1.610 | 9.236 | 11.070 | 12.832 | 15.085 | 16.748 |
| 6 | 0.676 | 0.872 | 1.237 | 1.635 | 2.204 | 10.645 | 12.592 | 14,440 | 16.812 | 18,548 |
| 7 | 0.989 | 1.239 | 1.690 | 2.167 | 2.833 | 12.017 | 14.067 | 16.012 | 18.474 | 20.276 |
| 8 | 1.344 | 1.646 | 2.180 | 2.733 | 3.490 | 13.362 | 15.507 | 17.534 | 20.090 | 21.954 |
| 9 | 1.735 | 2.088 | 2.700 | 3.325 | 4.168 | 14,684 | 16.919 | 19.022 | 21.665 | 23.587 |
| 10 | 2.156 | 2.558 | 3.247 | 3.940 | 4.865 | 15.987 | 18.307 | 20.483 | 23.209 | 25.188 |
| 11 | 2.603 | 3.053 | 3.816 | 4.575 | 5.578 | 17.275 | 19.675 | 21.920 | 24.724 | 26.755 |
| 12 | 3.074 | 3.571 | 4.404 | 5.226 | 6,304 | 18.549 | 21.026 | 23,337 | 26,217 | 28.300 |
| 13 | 3.565 | 4.107 | 5.009 | 5.892 | 7.041 | 19.812 | 22.362 | 24.735 | 27.687 | 29.817 |
| 14 | 4.075 | 4.660 | 5.629 | 6.571 | 7.790 | 21.064 | 23.685 | 26.119 | 29.141 | 31.319 |
| 15 | 4.600 | 5.229 | 6.262 | 7.261 | 8.547 | 22.307 | 24.996 | 27,488 | 30.577 | 32.799 |
| | | 5.812 | 6.908 | 7.962 | 9.312 | 23.542 | 26.296 | 28.845 | 32.000 | 34.267 |
| 16 | 5.142 | 6.407 | 7.564 | 8.682 | 10.085 | 24.769 | 27.587 | 30.190 | 33.408 | 35.716 |
| 17 | 5.697 | 7.015 | 8.231 | 9.390 | 10.865 | 25.989 | 28.869 | 31.526 | 34,805 | 37.156 |
| 18 | 6.265 | 7.632 | 8.906 | 10.117 | 11.651 | 27.203 | 30.143 | 32.852 | 36.190 | 38.580 |
| 19 | 6.843 7.434 | 8.260 | 9.591 | 10.851 | 12.443 | 28.412 | 31.410 | 34.170 | 37.566 | 39.997 |
| | | 8.897 | 10.283 | 11.591 | 13.240 | 29.615 | 32.670 | 35,478 | 38.930 | 41.399 |
| 21 | 8.033 | 9.542 | 10.283 | 12.338 | 14.042 | 30.813 | 33.924 | 36.781 | 40.289 | 42.796 |
| 22 | 8.643 | 10.195 | 11.688 | 13.090 | 14.848 | 32.007 | 35.172 | 38.075 | 41.637 | 44.179 |
| 23 | 9.260 | 10.193 | 12.401 | 13.848 | 15.659 | 33.196 | 36.415 | 39.364 | 42.980 | 45.558 |
| 24 25 | 9.886 10.519 | 11.523 | 13.120 | 14.611 | 16.473 | 34,381 | 37.652 | 40,646 | 44.313 | 46.925 |
| | | 12.198 | 13.844 | 15.379 | 17,292 | 35,563 | 38.885 | 41.923 | 45.642 | 48.290 |
| 26 | 11.160 | 12.198 | 14.573 | 16.151 | 18.114 | 26.741 | 40.113 | 43.194 | 46.962 | 49.642 |
| 27 | 11.807 | 13.565 | 15.308 | 16.928 | 18.939 | 37.916 | 41.337 | 44.461 | 48.278 | 50.993 |
| 28 | 12.461 | 14.256 | 16.147 | 17.708 | 19.768 | 39.087 | 42,557 | 45.772 | 49.586 | 52.333 |
| 29 30 | 13.120 | 14.954 | 16.791 | 18.493 | 20.599 | 40.256 | 43.773 | 46.979 | 50.892 | 53.672 |
| | | | | 19.280 | 21.433 | 41.422 | 44.985 | 48.231 | 52.190 | 55.000 |
| 31 | 14.457 | 15.655 | 17.538 | 20.072 | 22,271 | 42.585 | 46.194 | 49.480 | 53,486 | 56.328 |
| 32 | 15.134 | 16.362 | 18.291 | 20.866 | 23.110 | 43.745 | 47.400 | 50.724 | 54.774 | 57.646 |
| 33 | 15.814 | 17.073 | 19.046 | 21.664 | 23.952 | 44.903 | 48.602 | 51.966 | 56.061 | 58.964 |
| 34 | 16.501 | 17.789 | 19.806 | | 24.796 | 46.059 | 49.802 | 53.203 | 57.340 | 60.272 |
| 35 | 17.191 | 18,508 | 20.569 | 22.465 | | | 50.998 | 54.437 | 58.619 | 61.581 |
| 36 | 17.887 | 19.233 | 21.336 | 23.269 | 25.643 | 47.212 | 52.192 | 55.667 | 59.891 | 62.880 |
| 37 | 18.584 | 19.960 | 22.105 | 24.075 | 26.492 | 48.363 | 53.384 | 56.896 | 61.162 | 64.181 |
| 38 | 19.289 | 20.691 | 22.878 | 24.884 | 27.343 | 49.513 | 54.572 | 58.119 | 62.426 | 65.473 |
| 39 | 19.994 | 21.425 | 23.654 | 25.695 | 28.196 | 50.660 | 55.758 | 59.342 | 63.691 | 66.766 |
| 40 | 20.706 | 22.164 | 24.433 | 26.509 | 29.050 | 51.805 | J.J. 1 JO | سقد ۳ الدواد الد | | · |

For $\nu > 40$, $\chi^2_{\alpha,\nu} = \nu \left(1 - \frac{2}{9\nu} + z_{\alpha} \sqrt{\frac{2}{9\nu}} \right)^3$